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Carnegie Mellon Univ.
Dept. of Computer Science
15-415/615 – DB Applications

Faloutsos & Pavlo
Lecture#11 (R&G ch. 11)
Hashing

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Outline

- (static) hashing
- extendible hashing
- linear hashing
- Hashing vs B-trees

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(Static) Hashing

Problem: "*find EMP record with ssn=123*"

What if disk space was free, and time was at premium?

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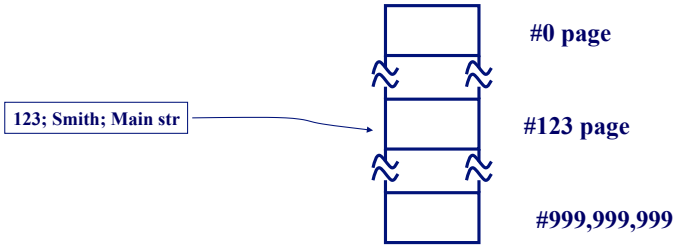
Hashing

A: Brilliant idea: key-to-address transformation:



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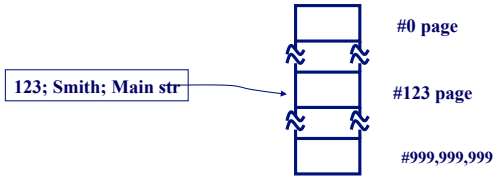
Hashing

Since space is NOT free:

- use *M*, instead of 999,999,999 slots
- hash function: *h(key) = slot-id*



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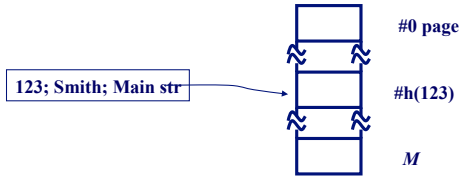
Hashing

Typically: each hash bucket is a page, holding many records:



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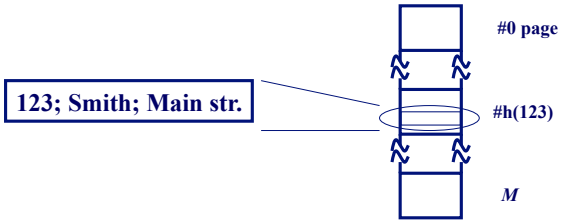
Hashing

Notice: could have **clustering**, or non-clustering versions:



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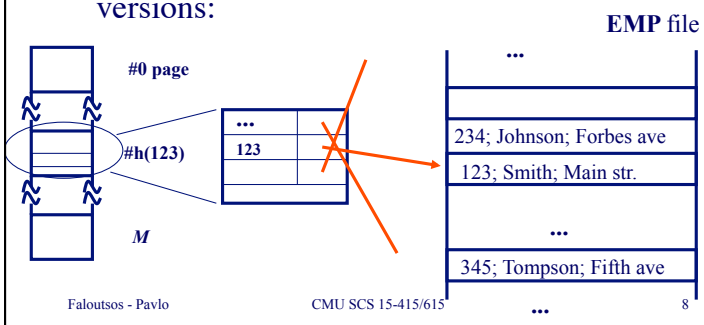
Hashing

Notice: could have clustering, or **non-clustering** versions:



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Indexing- overview

- hashing
  - hashing functions
  - size of hash table
  - collision resolution
- extendible hashing
- Hashing vs B-trees

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Design decisions

1) formula *h()* for hashing function
2) size of hash table *M*
3) collision resolution method

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Design decisions - functions

- Goal: uniform spread of keys over hash buckets
- Popular choices:
  - Division hashing
  - Multiplication hashing

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Division hashing

$$h(x) = (a*x+b) \mod M$$

- eg., h(ssn) = (ssn) mod 1,000
  - gives the last three digits of ssn
- M: size of hash table - choose a prime number, defensively (why?)

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Division hashing

- eg., *M*=2; hash on driver-license number (dln), where last digit is 'gender' (0/1 = M/F)
- in an army unit with predominantly male soldiers
- Thus: avoid cases where *M* and keys have common divisors - prime *M* guards against that!

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Multiplication hashing

h(x) = [fractional-part-of (x * φ)] * M

- ***φ***: golden ratio (0.618... = (sqrt(5)-1)/2)
- in general, we need an irrational number
- advantage: *M* need not be a prime number
- but ***φ*** must be irrational

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Other hashing functions

- quadratic hashing (bad)
- ...

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Other hashing functions

- quadratic hashing (bad)
- ...
- conclusion: use division hashing

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Design decisions

1) formula *h()* for hashing function
2) size of hash table *M*
3) collision resolution method

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Size of hash table

- eg., 50,000 employees, 10 employee-records / page
- Q: *M*=?? pages/buckets/slots

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Size of hash table

- eg., 50,000 employees, 10 employees/page
- Q: *M*=?? pages/buckets/slots
- A: utilization ~ 90% and
  - *M*: prime number

Eg., in our case: *M*= closest prime to 50,000/10 / 0.9 = 5,555

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Design decisions

1) formula *h()* for hashing function
2) size of hash table *M*
3) collision resolution method

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Collision resolution

- Q: what is a 'collision'?
- A: ??

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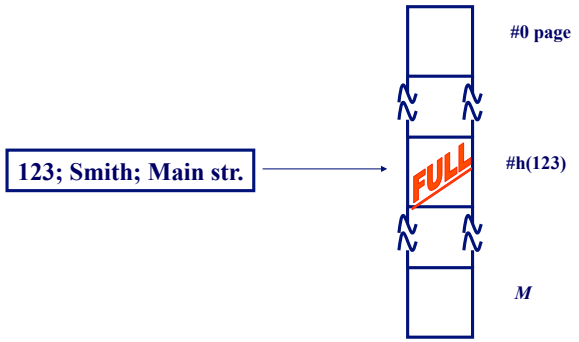
Collision resolution



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Collision resolution

- Q: what is a 'collision'?
- A: ??
- Q: why worry about collisions/overflows? (recall that buckets are ~90% full)
- A: 'birthday paradox'

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Collision resolution

- open addressing
  - linear probing (ie., put to next slot/bucket)
  - re-hashing
- separate chaining (ie., put links to overflow pages)

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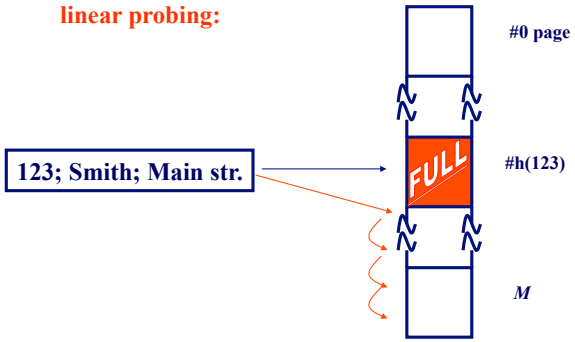
Collision resolution

linear probing:



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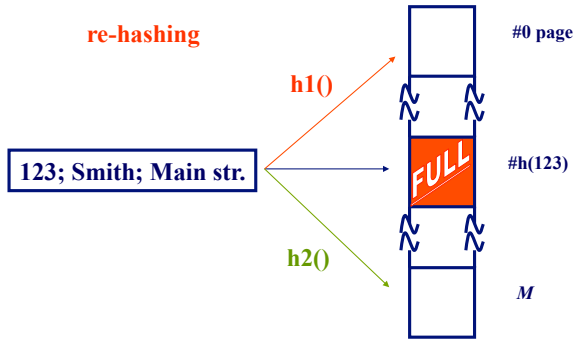
Collision resolution

re-hashing



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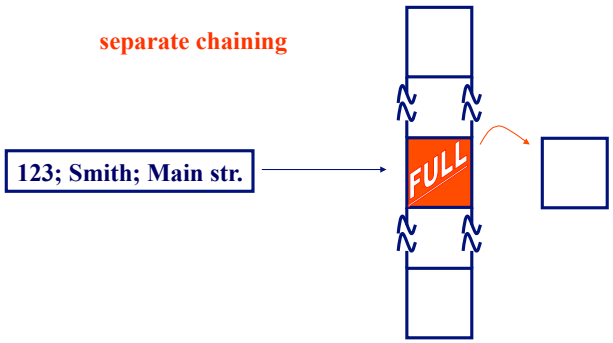
Collision resolution

separate chaining



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Design decisions - conclusions

- function: division hashing
  - *h(x) = (a*x+b) mod M*
- size *M*: ~90% util.; prime number.
- collision resolution: separate chaining
  - easier to implement (deletions!);
  - no danger of becoming full

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Outline

- (static) hashing
- extendible hashing
- linear hashing
- Hashing vs B-trees

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Problem with static hashing

- problem: overflow?
- problem: underflow? (underutilization)

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Solution: Dynamic/extendible hashing

- idea: shrink / expand hash table on demand..
- ..dynamic hashing

Details: how to grow gracefully, on overflow?

Many solutions - One of them: 'extendible hashing' [Fagin et al]

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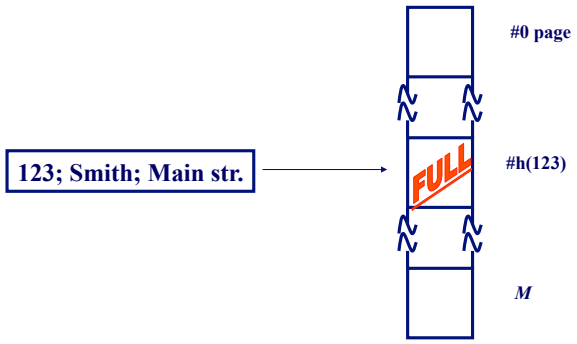
Extendible hashing



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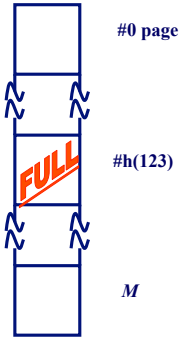
Extendible hashing

solution: don't overflow – instead:

SPLIT the bucket in two

123; Smith; Main str.



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Extendible hashing

in detail:

- keep a directory, with ptrs to hash-buckets
- Q: how to divide contents of bucket in two?
- A: hash each key into a very long bit string; keep only as many bits as needed

Eventually:

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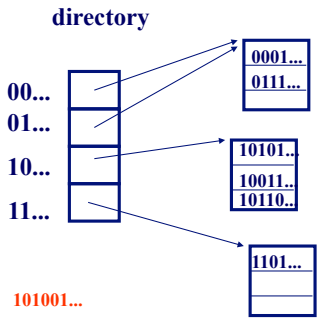
Extendible hashing



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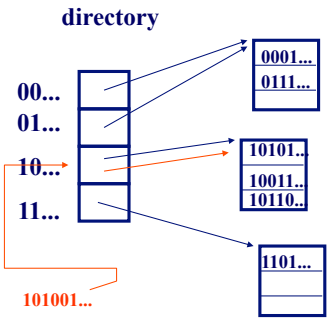
Extendible hashing



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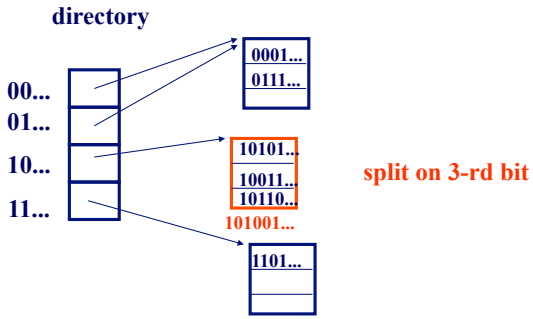
Extendible hashing



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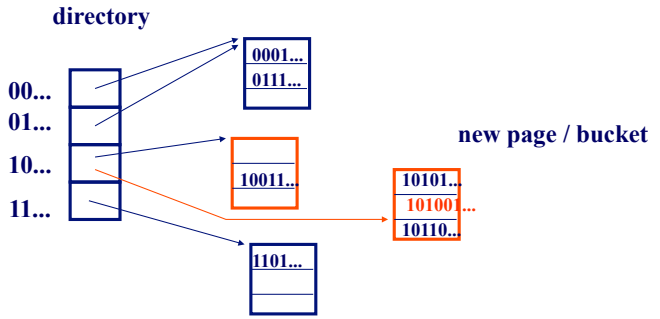
Extendible hashing



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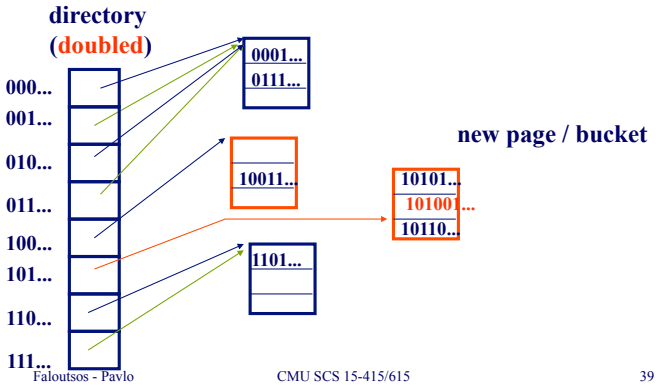
Extendible hashing



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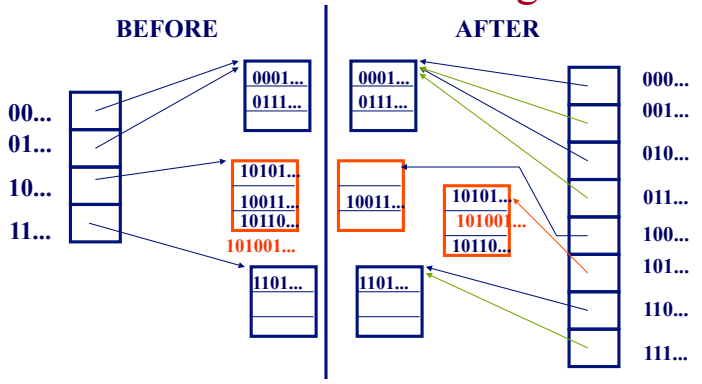
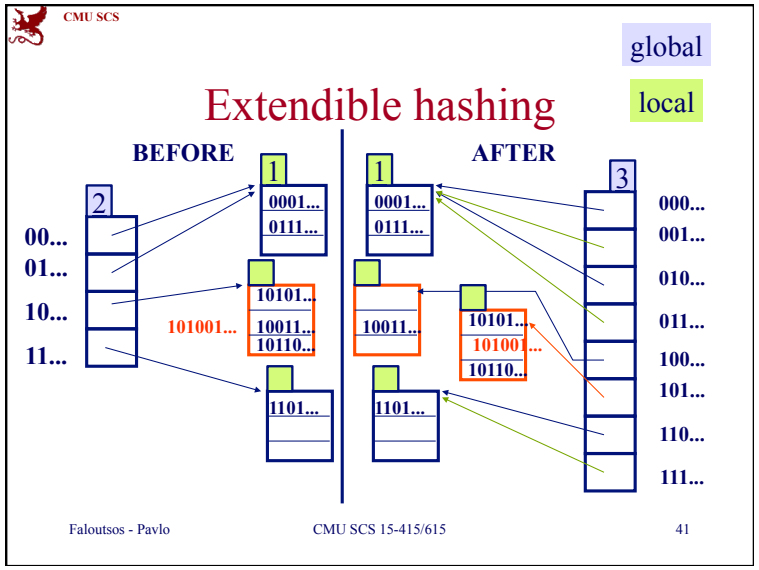
Extendible hashing

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global
local
Extendible hashing
BEFORE
AFTER
00...
01...
10...
11...
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0111...
10101...
10011...
10110...
101001...
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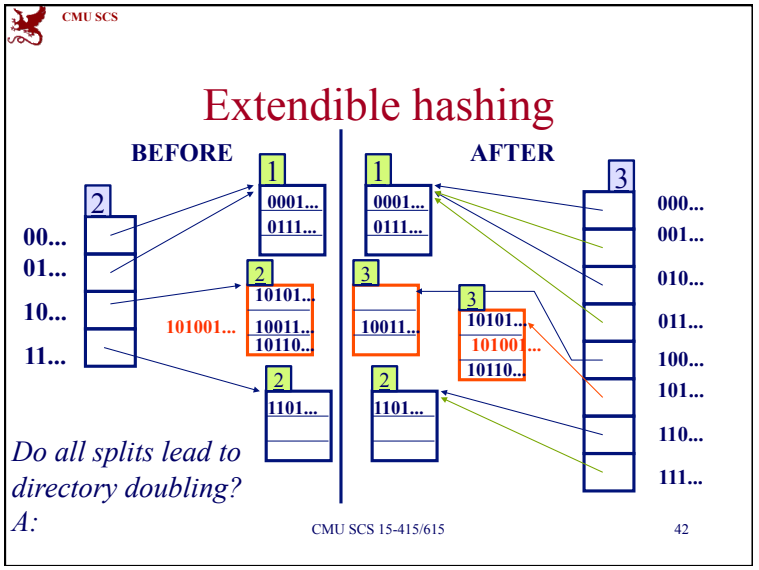
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Extendible hashing
BEFORE
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Do all splits lead to directory doubling?
A:
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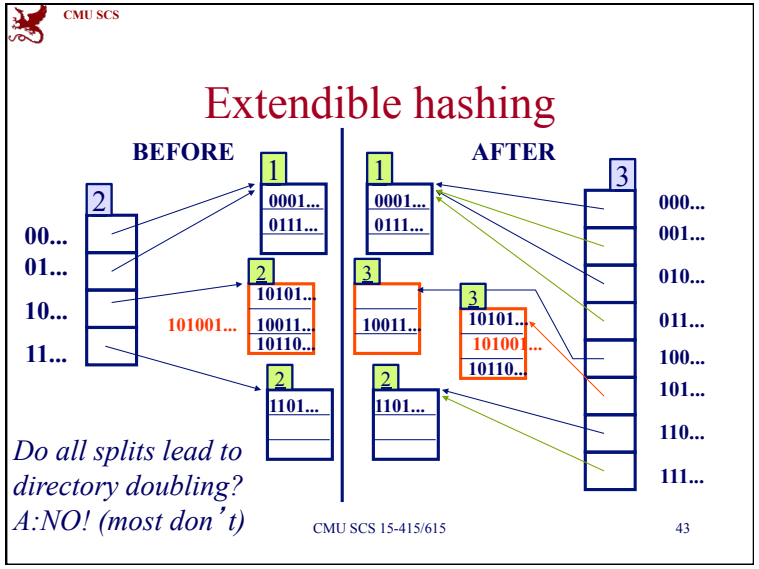
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Extendible hashing
BEFORE
AFTER
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10101...
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Do all splits lead to directory doubling?
A:NO! (most don't)
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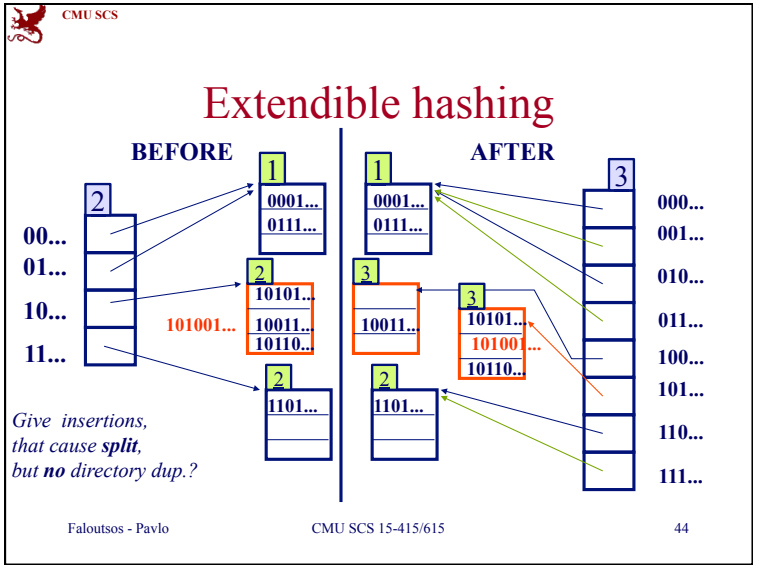
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Extendible hashing
BEFORE
AFTER
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101...
110...
111...
Give insertions, that cause split, but no directory dup.?
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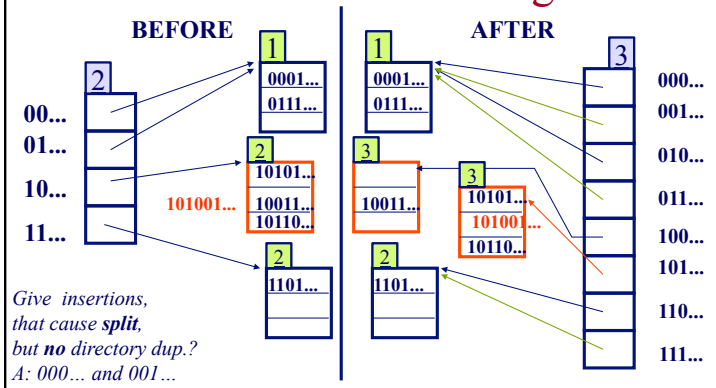
Extendible hashing



*Give insertions, that cause **split**, but **no** directory dup.? A: 000… and 001…*

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Extendible hashing

- Summary: directory doubles on demand
- or halves, on shrinking files
- needs 'local' and 'global' depth

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Outline

- (static) hashing
- extendible hashing
- linear hashing
- Hashing vs B-trees

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Linear hashing - overview

- Motivation
- main idea
- search algo
- insertion/split algo
- deletion

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Linear hashing

Motivation: ext. hashing needs directory etc etc; which doubles (ouch!)

Q: can we do something simpler, with smoother growth?

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Linear hashing

Motivation: ext. hashing needs directory etc etc; which doubles (ouch!)

Q: can we do something simpler, with smoother growth?

A: split buckets from left to right, **regardless** of which one overflowed ('crazy' , but it works well!) - Eg.:

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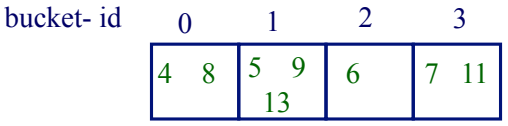
Linear hashing

Initially: *h(x) = x mod N* (N=4 here)

Assume capacity: 3 records / bucket

Insert key '17'



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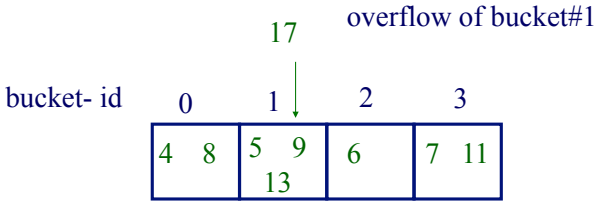
Linear hashing

Initially: *h(x) = x mod N* (N=4 here)



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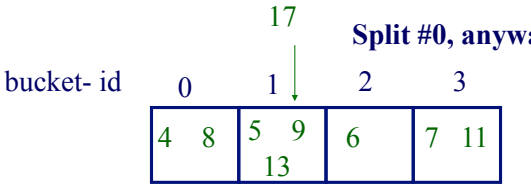
Linear hashing

Initially: *h(x) = x mod N* (N=4 here)

overflow of bucket#1

Split #0, anyway!!!



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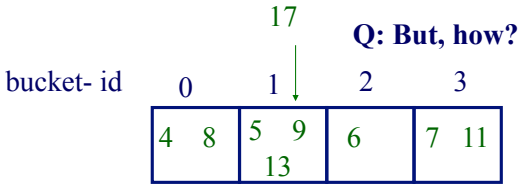
Linear hashing

Initially: *h(x) = x mod N* (N=4 here)

Split #0, anyway!!!

Q: But, how?



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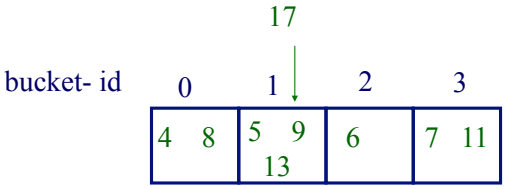
Linear hashing

A: use two h.f.: *h0(x) = x mod N*

*h1(x) = x mod (2*N)*



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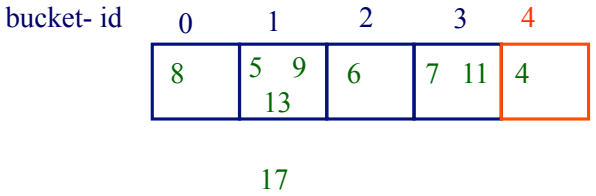
Linear hashing - after split:

A: use two h.f.: *h0(x) = x mod N*

*h1(x) = x mod (2*N)*



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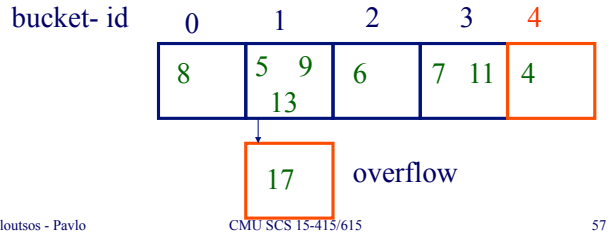
Linear hashing - after split:

A: use two h.f.: $h0(x) = x \bmod N$

$h1(x) = x \bmod (2*N)$



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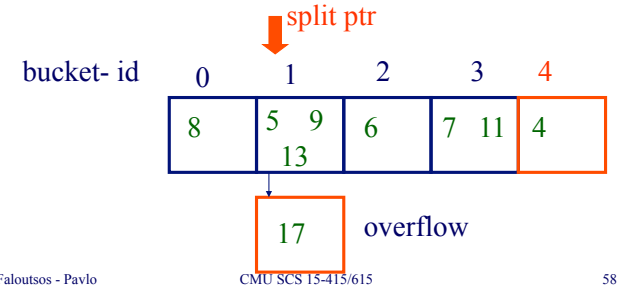
Linear hashing - after split:

A: use two h.f.: $h0(x) = x \bmod N$

$h1(x) = x \bmod (2*N)$



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Linear hashing - overview

- Motivation
- main idea
- search algo
- insertion/split algo
- deletion

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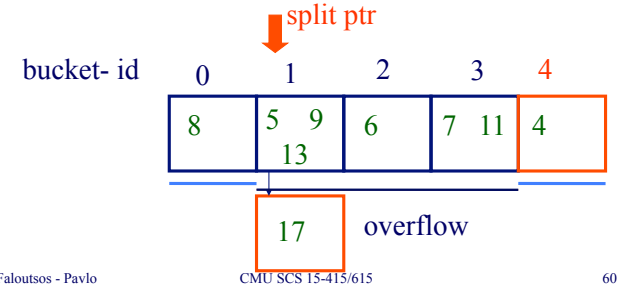
Linear hashing - searching?

- $h0(x) = x \bmod N$ *(for the un-split buckets)*
- $h1(x) = x \bmod (2*N)$ *(for the splitted ones)*



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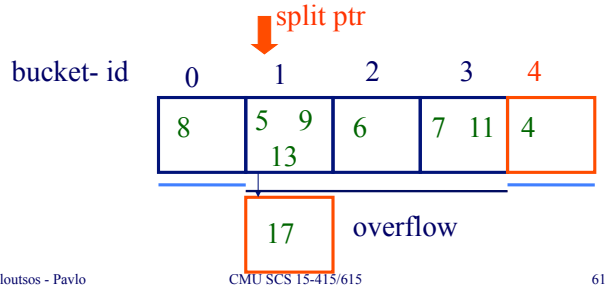
Linear hashing - searching?

Q1: find key ‘6’ ? Q2: find key ‘4’ ?

Q3: key ‘8’ ?



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Linear hashing - searching?

Algo to find key ‘k’ :

compute *b= h0(k);* *// original slot*

if *b<split-ptr* *// has been split*

compute *b=h1(k)*

search bucket *b*

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Linear hashing - overview

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Linear hashing - insertion?

Algo: insert key ‘*k*’

- compute appropriate bucket ‘*b*’
- if the **overflow criterion** is true
  - split the bucket of ‘split-ptr’
  - split-ptr ++ (*)

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Linear hashing - insertion?

notice: overflow criterion is up to us!!

Q: suggestions?

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Linear hashing - insertion?

notice: overflow criterion is up to us!!

Q: suggestions?

A1: space utilization >= u-max

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Linear hashing - insertion?

notice: overflow criterion is up to us!!

Q: suggestions?

A1: space utilization >= u-max

A2: avg length of ovf chains > max-len

A3: ....

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Linear hashing - insertion?

Algo: insert key '*k*'

- compute appropriate bucket '*b*'
- if the **overflow criterion** is true
  - split the bucket of 'split-ptr'
  - split-ptr ++ **(*)**

what if we reach the right edge??

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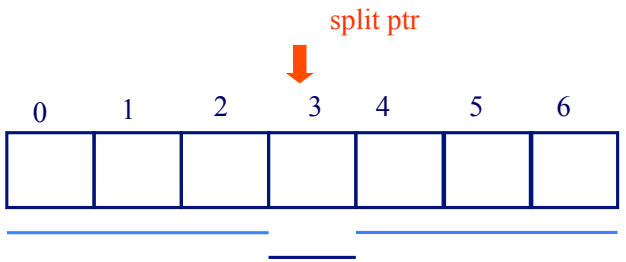
Linear hashing - split now?

h0(x) = x mod N (for the un-split buckets)
*h1(x) = x mod (2*N) for the splitted ones)*



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Linear hashing - split now?

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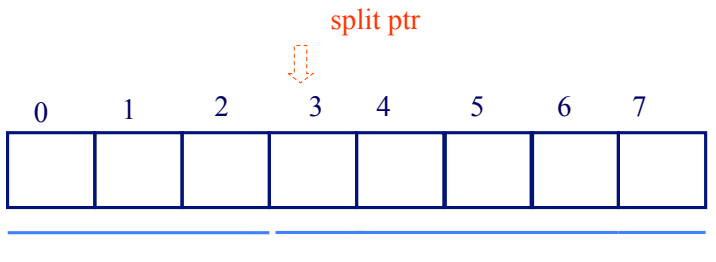
*h1(x) = x mod (2*N) (for the splitted ones)*



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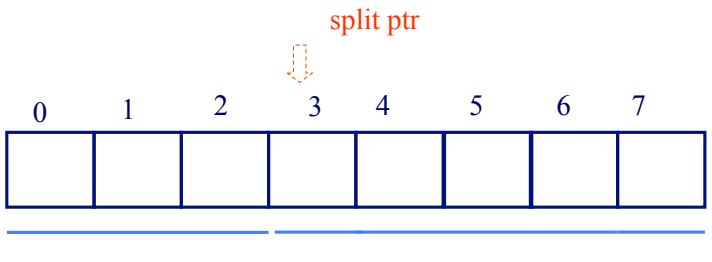
Linear hashing - split now?

~~*h0(x) = x mod N (for the un-split buckets)*~~
*h1(x) = x mod (2*N) (for the splitted ones)*



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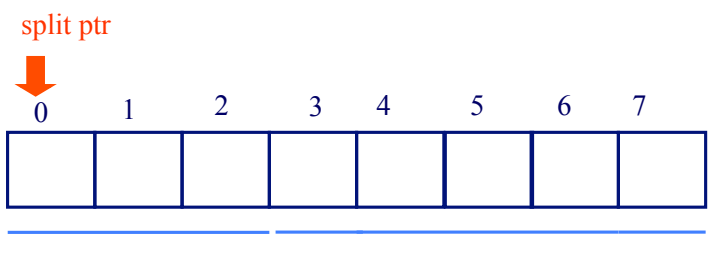
Linear hashing - split now?

~~*h0(x) = x mod N (for the un-split buckets)*~~
*h1(x) = x mod (2*N) (for the splitted ones)*



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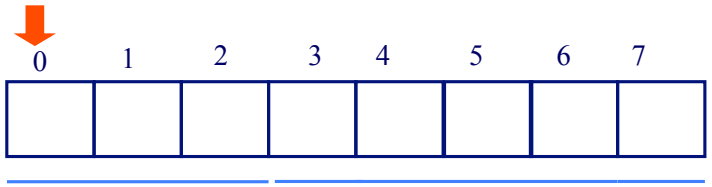
Linear hashing - split now?

this state is called **'full expansion'**

split ptr



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Linear hashing - observations

In general, at any point of time, we have at **most two** h.f. active, of the form:

- $h_n(x) = x \bmod (N * 2^n)$
- $h_{n+1}(x) = x \bmod (N * 2^{n+1})$

(after a full expansion, we have only one h.f.)

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Linear hashing - overview

- Motivation
- main idea
- search algo
- insertion/split algo
- deletion

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Linear hashing - deletion?

- reverse of insertion:

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Linear hashing - deletion?

- reverse of insertion:
- if the underflow criterion is met
  - contract!

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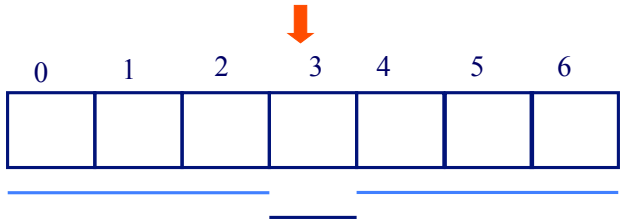
Linear hashing - how to contract?

h0(x) = mod N *(for the un-split buckets)*
*h1(x) = mod (2*N)* *(for the splitted ones)*

split ptr



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Linear hashing - how to contract?

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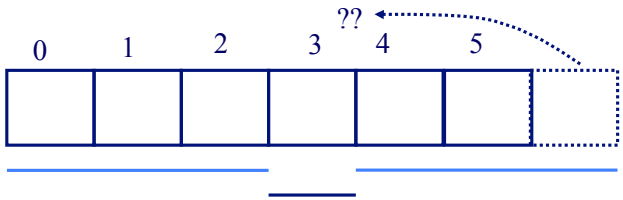

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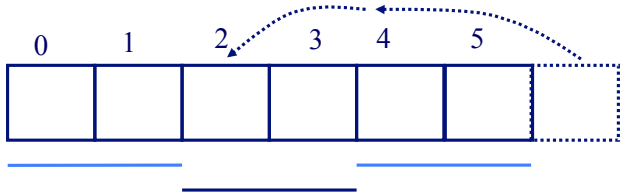

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Linear hashing - how to contract?

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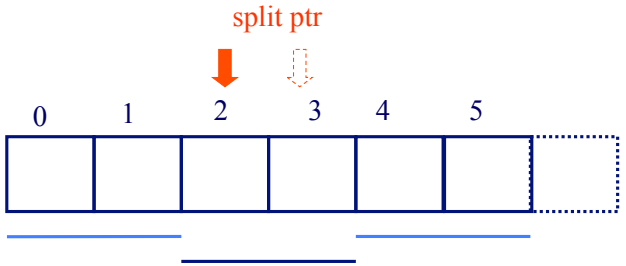

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Linear hashing - how to contract?

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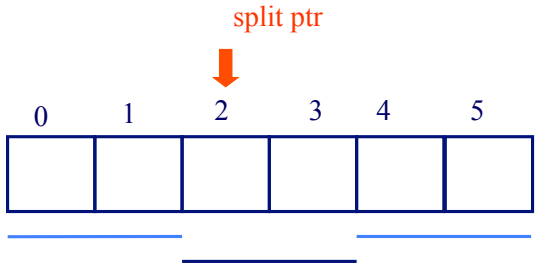

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Outline

- (static) hashing
- extendible hashing
- linear hashing
- Hashing vs B-trees

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Hashing - pros?

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Hashing - pros?

- Speed,
  - on exact match queries
  - on the average

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B(+)-trees - pros?

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B(+)-trees - pros?

- Speed on search:
  - exact match queries, worst case
  - range queries
  - nearest-neighbor queries
- Speed on insertion + deletion
- smooth growing and shrinking (no re-org)

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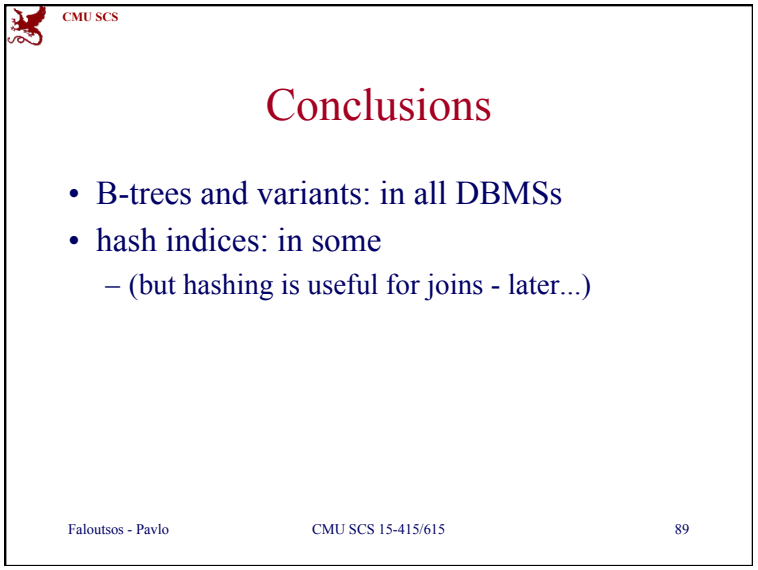
B(+)-trees vs Hashing

- Speed on search:
  - exact match queries, worst case
  - range queries
  - nearest-neighbor queries
- Speed on insertion + deletion
- smooth growing and shrinking (no re-org)

- Speed,
  - on exact match queries
  - on the average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Conclusions
• B-trees and variants: in all DBMSs
• hash indices: in some
– (but hashing is useful for joins - later...)
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