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Carnegie Mellon Univ.
Dept. of Computer Science
15-415/615 – DB Applications

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Lecture#5: *Relational calculus*




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General Overview - rel. model

- history
- concepts
- Formal query languages
 - relational algebra
 - **rel. tuple calculus**
 - rel. domain calculus

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Overview - detailed

- rel. tuple calculus
 - why?
 - details
 - examples
 - equivalence with rel. algebra
 - more examples; ‘safety’ of expressions
- rel. domain calculus + QBE

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Motivation

- Q: weakness of rel. algebra?
- A: procedural
 - describes the steps (ie., ‘**how**’)
 - (still useful, for query optimization)

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Solution: rel. calculus

- describes **what** we want
- two equivalent flavors: 'tuple' and 'domain' calculus
- basis for SQL and QBE, resp.
- Useful for proofs (see query optimization, later)

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Rel. tuple calculus (RTC)


- first order logic

$$\{t \mid P(t)\}$$

'Give me tuples 't', satisfying predicate P - eg:

$$\{t \mid t \in STUDENT\}$$

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Details

- symbols allowed:
 - $\wedge, \vee, \neg, \Rightarrow$
 - $>, <, =, \neq, \leq, \geq,$
 - $(,), \in$
- quantifiers \forall, \exists

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Specifically

- Atom
 - $t \in TABLE$
 - $t.attr \leq const$
 - $t.attr \leq s.attr'$

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Specifically

- Formula:
 - atom
 - if P1, P2 are formulas, so are $P1 \wedge P2; P1 \vee P2 \dots$
 - if P(s) is a formula, so are $\exists s(P(s))$
 $\forall s(P(s))$

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Specifically

- Reminders:
 - DeMorgan $P1 \wedge P2 \equiv \neg(\neg P1 \vee \neg P2)$
 - implication: $P1 \Rightarrow P2 \equiv \neg P1 \vee P2$
 - double negation:
 - $\forall s \in TABLE (P(s)) \equiv \neg \exists s \in TABLE (\neg P(s))$

‘every human is mortal : no human is immortal’

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Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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Examples

- find all student records

$\{t \mid t \in STUDENT\}$

output tuple of type ‘STUDENT’

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(Goal: evidence that RTC = RA)

- Full proof: complicated
- We'll just show examples of the 5 RA fundamental operators, and how RTC can handle them
- (Quiz: which are the 5 fundamental op's?)

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FUNDAMENTAL Relational operators

• selection	$\sigma_{condition} (R)$
• projection	$\pi_{att-list} (R)$
• cartesian product	MALE x FEMALE
• set union	$R \cup S$
• set difference	$R - S$

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Examples

- (selection) find student record with ssn=123

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Examples

- ✓ σ selection
- π projection
- X cartesian product
- U set union
- - set difference

- (selection) find student record with ssn=123

$$\{t \mid t \in STUDENT \wedge t.ssn = 123\}$$

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Examples

- (projection) find **name** of student with **ssn=123**

$$\{t \mid t \in \text{STUDENT} \wedge t.ssn = 123\}$$

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- ✓ σ selection
- ✓ π projection
- X cartesian product
- U set union
- - set difference

Examples

- (projection) find name of student with **ssn=123**

$$\{t \mid \exists s \in \text{STUDENT}(s.ssn = 123 \wedge t.name = s.name)\}$$

↑
‘t’ has only one column

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‘Tracing’

$$\{t \mid \exists s \in \text{STUDENT}(s.ssn = 123 \wedge t.name = s.name)\}$$

Name
aaaa
...
jones
...
zzzz

\xrightarrow{s}

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

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- ✓ σ selection
- ✓ π projection
- X cartesian product
- ✓ U set union
- - set difference

Examples cont’d

- (union) get records of both PT and FT students

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Examples cont'd

- (union) get records of both PT and FT students

$$\{t \mid t \in FT_STUDENT \vee t \in PT_STUDENT\}$$

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Examples

- ✓ σ selection
- ✓ π projection
- X cartesian product
- ✓ U set union
- ✓ - set difference

- difference: find students that are not staff

(assuming that STUDENT and STAFF are union-compatible)

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Examples

- difference: find students that are not staff

$$\{t \mid t \in STUDENT \wedge t \notin STAFF\}$$

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Cartesian product

- eg., dog-breeding: MALE x FEMALE
- gives all possible couples

MALE		FEMALE	
name	x	name	=
spike		lassie	
spot		shiba	
		M.name	F.name
		spike	lassie
		spike	shiba
		spot	lassie
		spot	shiba

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Cartesian product

- ✓ σ selection
- ✓ π projection
- ✓ \times cartesian product
- ✓ \cup set union
- ✓ $-$ set difference

- find all the pairs of (male, female)

$$\{t \mid \exists m \in MALE \wedge \exists f \in FEMALE \\ t.m - name = m.name \wedge t.f - name = f.name\}$$

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'Proof' of equivalence

- rel. algebra \leftrightarrow rel. tuple calculus

- ✓ σ selection
- ✓ π projection
- ✓ \times cartesian product
- ✓ \cup set union
- ✓ $-$ set difference

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Overview - detailed

- rel. tuple calculus
 - why?
 - details
 - examples
 - equivalence with rel. algebra
 - **more examples**; 'safety' of expressions
- re. domain calculus + QBE

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More examples

- join: find names of students taking 15-415

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Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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More examples

- join: find names of students taking 15-415

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c - id = 15 - 415)\}$$

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More examples

- join: find names of students taking 15-415

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c - id = 15 - 415)\}$$

join

projection

selection

(Remember: 'SPJ')

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More examples

- 3-way join: find names of students taking a 2-unit course

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Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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More examples

- 3-way join: find names of students taking a 2-unit course

$$\{t \mid \exists s \in STUDENT \wedge \exists e \in TAKES$$

$$\exists c \in CLASS (s.ssn = e.ssn \wedge$$

$$e.c-id = c.c-id \wedge$$

$$t.name = s.name \wedge$$

$$c.units = 2)\}$$

join
projection
selection

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More examples

- 3-way join: find names of students taking a 2-unit course - in rel. algebra??

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More examples

- 3-way join: find names of students taking a 2-unit course - in rel. algebra??

$$\pi_{name}(\sigma_{units=2}(STUDENT \bowtie TAKES \bowtie CLASS))$$

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Even more examples:

- self -joins: find Tom's grandparent(s)

PC		PC	
p-id	c-id	p-id	c-id
Mary	Tom	Mary	Tom
Peter	Mary	Peter	Mary
John	Tom	John	Tom

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Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{t \mid \exists p \in PC \wedge \exists q \in PC$$

$$(p.c - id = q.p - id \wedge$$

$$p.p - id = t.p - id \wedge$$

$$q.c - id = "Tom")\}$$

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Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

SHIPMENT		ABOMB		BAD_S	
s#	p#	p#	s#	s#	p#
s1	p1	p1	s1	s1	p1
s2	p1	p1	s1		
s1	p2	p2			
s3	p1	p1			
s5	p3	p3			

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Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts


$$\{t \mid \forall p (p \in ABOMB \Rightarrow ($$

$$\exists s \in SHIPMENT ($$

$$t.s\# = s.s\# \wedge$$

$$s.p\# = p.p\#))\}$$


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General pattern

- three equivalent versions:
 - 1) if it's bad, he shipped it
 $\{t \mid \forall p(p \in ABOMB \Rightarrow (P(t)))\}$
 - 2) either it was good, or he shipped it
 $\{t \mid \forall p(p \notin ABOMB \vee (P(t)))\}$
 - 3) there is no bad shipment that he missed
 $\{t \mid \neg \exists p(p \in ABOMB \wedge (\neg P(t)))\}$

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General pattern

- three equivalent versions:
 - 1) if it's bad, he shipped it
 $\{t \mid \forall p(p \in ABOMB \Rightarrow (P(t)))\}$
 - 2) either it was good, or he shipped it
 $\{t \mid \forall p(p \notin ABOMB \vee (P(t)))\}$
 - 3) there is no bad shipment that he missed
 $\{t \mid \neg \exists p(p \in ABOMB \wedge (\neg P(t)))\}$

$a \Rightarrow b$
 $\neg a \vee b$

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General pattern

- three equivalent versions:
 - 1) if it's bad, he shipped it
 $\{t \mid \forall p(p \in ABOMB \Rightarrow (P(t)))\}$
 - 2) either it was good, or he shipped it
 $\{t \mid \forall p(p \notin ABOMB \vee (P(t)))\}$
 - 3) there is no bad shipment that he missed
 $\{t \mid \neg \exists p(p \in ABOMB \wedge (\neg P(t)))\}$

$\forall x(P(x)) = \neg \exists x(\neg P(x))$

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a ⇒ b is the same as ¬a ∨ b

		b	
		T	F
a	T	T	F
	F	T	T

- If a is true, b must be true for the implication to be true. If a is true and b is false, the implication evaluates to false.
- If a is not true, we don't care about b, the expression is always true.

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More on division

- find (SSNs of) students that take all the courses that ssn=123 does (and maybe even more)
find students 's' so that
if 123 takes a course => so does 's'

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More on division

- find students that take all the courses that ssn=123 does (and maybe even more)

$$\{o \mid \forall t((t \in TAKES \wedge t.ssn = 123) \Rightarrow \exists t1 \in TAKES(t1.c-id = t.c-id \wedge t1.ssn = o.ssn))\}$$

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Safety of expressions

- FORBIDDEN: ~~$\{t \mid t \notin STUDENT\}$~~

It has infinite output!!

- Instead, always use

$$\{t \mid \dots t \in SOME-TABLE\}$$


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Overview - conclusions

- rel. tuple calculus: DECLARATIVE
 - dfn
 - details
 - equivalence to rel. algebra
- rel. domain calculus + QBE**

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General Overview

- relational model
- Formal query languages
 - relational algebra
 - rel. tuple calculus
 - **rel. domain calculus**

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Rel. domain calculus (RDC)

- Q: why?
- A: slightly easier than RTC, although equivalent - basis for QBE.
- idea: domain variables (w/ F.O.L.) - eg:
- ‘find STUDENT record with ssn=123’

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Rel. Dom. Calculus

- find STUDENT record with ssn=123’

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge s = 123 \}$$

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Details

- Like R.T.C - symbols allowed:
 - $\wedge, \vee, \neg, \Rightarrow$
 - $>, <, =, \neq, \leq, \geq$
 - $(,), \in$
- quantifiers \forall, \exists

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Details

- but: domain (= column) variables, as opposed to tuple variables, eg:

$$\langle s, n, a \rangle \in STUDENT$$

ssn $name$ $address$

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Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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Examples

- find all student records

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \}$$

RTC: $\{ t \mid t \in STUDENT \}$


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Examples

- (selection) find student record with ssn=123

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('Proof' of RDC = RA)

- Again, we show examples of the 5 fundamental operators, in RDC

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Examples

- (selection) find student record with ssn=123

RTC: $\{t \mid t \in STUDENT \wedge t.ssn = 123\}$

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Examples

- (selection) find student record with ssn=123


$$\{ \langle 123, n, a \rangle \mid \langle 123, n, a \rangle \in STUDENT \}$$

or

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge s = 123 \}$$

RTC: $\{t \mid t \in STUDENT \wedge t.ssn = 123\}$

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Examples

- (projection) find name of student with ssn=123

$$\{ \langle n \rangle \mid \langle 123, n, a \rangle \in STUDENT \}$$

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Examples

- (projection) find name of student with ssn=123

$$\{ \langle n \rangle \mid \exists a (\langle 123, n, a \rangle \in STUDENT) \}$$

↑
need to 'restrict' "a"

RTC: $\{ t \mid \exists s \in STUDENT (s.ssn = 123 \wedge t.name = s.name) \}$

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Examples cont'd

- (union) get records of both PT and FT students

RTC: $\{ t \mid t \in FT_STUDENT \vee t \in PT_STUDENT \}$

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Examples cont'd

- (union) get records of both PT and FT students

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in FT_STUDENT \vee \langle s, n, a \rangle \in PT_STUDENT \}$$

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Examples

- difference: find students that are not staff

RTC: $\{ t \mid t \in STUDENT \wedge t \notin STAFF \}$

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Examples

- difference: find students that are not staff

$$\{ \langle s, n, a \rangle \mid \langle s, n, a \rangle \in STUDENT \wedge \langle s, n, a \rangle \notin STAFF \}$$

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Cartesian product

- eg., dog-breeding: MALE x FEMALE
- gives all possible couples

MALE	x	FEMALE	=	M.name	F.name
name		name		spike	lassie
spike		lassie		spike	shiba
spot		shiba		spot	lassie
				spot	shiba

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Cartesian product

- find all the pairs of (male, female) - RTC:

$$\{ t \mid \exists m \in MALE \wedge \exists f \in FEMALE \wedge t.m - name = m.name \wedge t.f - name = f.name \}$$


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Cartesian product

- find all the pairs of (male, female) - RDC:

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Cartesian product

- find all the pairs of (male, female) - RDC:

$$\{ \langle m, f \rangle \mid \langle m \rangle \in \text{MALE} \wedge \langle f \rangle \in \text{FEMALE} \}$$

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'Proof' of equivalence

- rel. algebra \leftrightarrow rel. domain calculus
- \leftrightarrow rel. tuple calculus

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Overview - detailed

- rel. domain calculus
 - why?
 - details
 - examples
 - equivalence with rel. algebra
 - **more examples**; 'safety' of expressions

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More examples

- join: find names of students taking 15-415

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Reminder: our Mini-U db

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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More examples

- join: find names of students taking 15-415 - in RTC

$$\{t \mid \exists s \in STUDENT$$

$$\wedge \exists e \in TAKES (s.ssn = e.ssn \wedge$$

$$t.name = s.name \wedge$$

$$e.c-id = 15-415)\}$$

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More examples

- join: find names of students taking 15-415 - in RDC

$$\{ \langle n \rangle \mid \exists s \exists a \exists g (\langle s, n, a \rangle \in STUDENT$$

$$\wedge \langle s, 15-415, g \rangle \in TAKES) \}$$

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Sneak preview of QBE:

$$\{ \langle n \rangle \mid \exists s \exists a \exists g (\langle s, n, a \rangle \in STUDENT$$

$$\wedge \langle s, 15-415, g \rangle \in TAKES) \}$$

STUDENT		
Ssn	Name	Address
<u>x</u>	P.	

TAKES		
SSN	c-id	grade
<u>x</u>	15-415	

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Sneak preview of QBE:

- very user friendly
- heavily based on RDC
- very similar to MS Access interface and pgAdminIII

STUDENT		
Ssn	Name	Address
<u>x</u>	P.	

TAKES		
SSN	c-id	grade
<u>x</u>	15-415	

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More examples

- 3-way join: find names of students taking a 2-unit course - in RTC:

$$\{t \mid \exists s \in STUDENT \wedge \exists e \in TAKES$$

$\exists c \in CLASS (s.ssn = e.ssn \wedge$	join
$e.c-id = c.c-id \wedge$	
$t.name = s.name \wedge$	projection
$c.units = 2)\}$	selection

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Reminder: our Mini-U db

_x	.P		_y	2
----	----	--	----	---

STUDENT		
Ssn	Name	Address
123	smith	main str
234	jones	forbes ave

CLASS		
c-id	c-name	units
15-413	s.e.	2
15-412	o.s.	2

TAKES		
SSN	c-id	grade
123	15-413	A
234	15-413	B

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More examples

- 3-way join: find names of students taking a 2-unit course

$$\{ \langle n \rangle \mid \dots \dots \dots$$

$$\langle s, n, a \rangle \in STUDENT \wedge$$

$$\langle s, c, g \rangle \in TAKES \wedge$$

$$\langle c, cn, 2 \rangle \in CLASS \}$$

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More examples

- 3-way join: find names of students taking a 2-unit course

$$\{ \langle n \rangle \mid \exists s, a, c, g, cn($$

$$\quad \langle s, n, a \rangle \in STUDENT \wedge$$

$$\quad \langle s, c, g \rangle \in TAKES \wedge$$

$$\quad \langle c, cn, 2 \rangle \in CLASS$$

$$\quad \left. \right\}$$

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Even more examples:

- self -joins: find Tom's grandparent(s)

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

PC	
p-id	c-id
Mary	Tom
Peter	Mary
John	Tom

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Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{ t \mid \exists p \in PC \wedge \exists q \in PC$$

$$\quad (p.c - id = q.p - id \wedge$$

$$\quad \quad p.p - id = t.p - id \wedge$$

$$\quad \quad q.c - id = "Tom") \}$$

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Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{ t \mid \exists p \in PC \wedge \exists q \in PC$$

$$\quad (p.c - id = q.p - id \wedge$$

$$\quad \quad p.p - id = t.p - id \wedge$$

$$\quad \quad q.c - id = "Tom") \}$$

$$\{ \langle g \rangle \mid \exists p (\langle g, p \rangle \in PC \wedge$$

$$\quad \langle p, "Tom" \rangle \in PC) \}$$

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Even more examples:

- self -joins: find Tom's grandparent(s)

$$\{ \langle g \rangle \mid \exists p (\langle g, p \rangle \in PC \wedge \langle p, "Tom" \rangle \in PC) \}$$

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Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

SHIPMENT	
s#	p#
s1	p1
s2	p1
s1	p2
s3	p1
s5	p3

÷

ABOMB
p#
p1
p2

=

BAD_S
s#
s1

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Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{ t \mid \forall p (p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#))) \}$$

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Hard examples: DIVISION

- find suppliers that shipped all the ABOMB parts

$$\{ t \mid \forall p (p \in ABOMB \Rightarrow (\exists s \in SHIPMENT (t.s\# = s.s\# \wedge s.p\# = p.p\#))) \} \quad \{ \langle s \rangle \mid \forall p (\langle p \rangle \in ABOMB \Rightarrow \langle s, p \rangle \in SHIPMENT) \}$$

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More on division

- find students that take all the courses that ssn=123 does (and maybe even more)

$$\{o \mid \forall t((t \in TAKES \wedge t.ssn = 123) \Rightarrow \exists t1 \in TAKES(t1.c - id = t.c - id \wedge t1.ssn = o.ssn))\}$$

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More on division

- find students that take all the courses that ssn=123 does (and maybe even more)

$$\{< s > \mid \forall c(\exists g(< 123, c, g > \in TAKES) \Rightarrow \exists g'(< s, c, g' > \in TAKES))\}$$

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Safety of expressions

- similar to RTC
- FORBIDDEN:

$$\{< s, n, a > \mid < s, n, a > \notin STUDENT\}$$

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Overview - detailed

- rel. domain calculus + QBE**
 - dfn
 - details
 - equivalence to rel. algebra

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Fun Drill: Your turn ...

- Schema:
 - Movie(title, year, studioName)
 - ActsIn(movieTitle, starName)
 - Star(name, gender, birthdate, salary)

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Your turn ...

- Queries to write in TRC:
 - Find all movies by Paramount studio
 - ... movies starring Kevin Bacon
 - Find stars who have been in a film w/Kevin Bacon
 - Stars within six degrees of Kevin Bacon*
 - Stars connected to K. Bacon via any number of films**

* Try *two* degrees for starters ** Good luck with this one!

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Answers ...

- Find all movies by Paramount studio

$$\{M \mid M \in \text{Movie} \wedge M.\text{studioName} = \text{'Paramount'}\}$$

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Answers ...

- Movies starring Kevin Bacon

$$\{M \mid M \in \text{Movie} \wedge \exists A \in \text{ActsIn}(A.\text{movieTitle} = M.\text{title} \wedge A.\text{starName} = \text{'Bacon'})\}$$

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Answers ...


- Stars who have been in a film w/Kevin Bacon

$$\{S \mid S \in \text{Star} \wedge \exists A \in \text{ActsIn}(A.\text{starName} = S.\text{name} \wedge \exists A2 \in \text{ActsIn}(A2.\text{movieTitle} = A.\text{movieTitle} \wedge A2.\text{starName} = \text{'Bacon'}))\}$$

S: name ...

A: movie star

A2: movie star 15



'Bacon'

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Answers ...

- Stars within ~~six~~^{two} degrees of Kevin Bacon

$$\{S \mid S \in \text{Star} \wedge \exists A \in \text{ActsIn}(A.\text{starName} = S.\text{name} \wedge \exists A2 \in \text{ActsIn}(A2.\text{movieTitle} = A.\text{movieTitle} \wedge \exists A3 \in \text{ActsIn}(A3.\text{starName} = A2.\text{starName} \wedge \exists A4 \in \text{ActsIn}(A4.\text{movieTitle} = A3.\text{movieTitle} \wedge A4.\text{starName} = \text{'Bacon'}))\}$$

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Two degrees:

S: name ...

A3: movie star

A4: movie star 15



'Bacon'

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Two degrees:


S: name ...

A: movie star

A2: movie star


A3: movie star

A4: movie star 15



'Bacon'

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Answers ...

- Stars connected to K. Bacon via any number of films
- **Sorry ... that was a trick question**
 - Not expressible in relational calculus!!
- **What about in relational algebra?**
 - No – RA, RTC, RDC are equivalent

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Expressive Power

- Expressive Power (Theorem due to Codd):
 - Every query that can be expressed in relational algebra can be expressed as a safe query in RDC / RTC; the converse is also true.
- Relational Completeness:
 - Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus. (actually, SQL is more powerful, as we will see...)

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Summary

- The relational model has rigorously defined query languages — simple and powerful.
- Relational algebra is more operational/**procedural**
 - useful as internal representation for query evaluation plans
- Relational calculus is **declarative**
 - users define queries in terms of what they want, not in terms of how to compute it.

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Summary - cnt'd

- Several ways of expressing a given query
 - a *query optimizer* chooses best plan.
- Algebra and safe calculus: same *expressive power*
 - => *relational completeness*.

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